Formulation of the Dynamic Characteristics of Fuel Cells

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Abstract

The dynamic voltage-current (V-I) characteristic of fuel cell stacks is important for designing an electrical system. The method presented in this paper is using a numerical method namely least squares technique to represent the electrochemical equations of fuel cells. The variation of the cell voltage with current should be optimized in order to maximize the power output that is the paramount criteria for all power conditioning designs. As power conditioning system under high frequency power conversion should also be required to accommodate for dynamic characteristics, the present model also aims for both steady-state and dynamic modelings for the design and optimization of the power system. The model includes thermodynamic potential and activation of the electrode, concentration voltage, ohmic voltage and other characteristic parameters. The numerical method has also used to examine a number of fuel cells. Experimental result has been used to validate the model and excellent agreement has been found. A simulation model has been developed and used to examine the validation of the model. It is confirmed that the fuel cell model is a simple and accurate method for design a fuel cell power conditioning and associated system.

Introduction

Due to high efficiency, low aggression to the environment, diversity of fuels, reusability of exhaust heat, and superior reliability and durability, fuel cells are expected to play a significant role in distributed generation. Consequently, there is an increasing interest in the use of fuel cell stack systems for both mobile and stationary applications including portable power, transportation vehicle, building cogeneration, uninterruptible power source and distributed power for utilities.

Steady-state voltage-current characteristics are an important performance of fuel cells and indicate the output feature of fuel cells. These characteristics are highly nonlinear and can be acquired through experiment or computed by a number of electrochemical equations (Correa, Mann and Pukrushpan). Analytically modeling steady-state voltage-current characteristics is beneficial for design of the power conditioning units, design of simulators of fuel cell stack systems, and design of system controllers. Hence, some attempts to analytically model steady-state characteristics of fuel cells were reported. In Acharya, et. al.'s study, the experimental steady-state V-I characteristics were fitted by the cubic polynomial. However, the fitting model seems to be not accurate enough because the highly nonlinearity of the steady-state voltage-current characteristics cannot be described by using cubic polynomials accurately. A fuel cell simulator was developed by Lee, et. al. For their simulator, the nonlinear steady-state voltage-current characteristics were modeled by using piecewise linear approach. Such an approach is simple and but its accuracy is not good. Xue, et. al. (2006) developed the unified model of steady-state and dynamic voltage-current characteristics. They used the one-dimensional least square polynomial to model the steady-state and dynamic V-I characteristics and hence the accuracy is good.

As we known, steady-state voltage-current characteristics of PEM fuel cells are affected by the temperature and the pressures of the hydrogen and oxygen. Generally, the pressures of the hydrogen and oxygen may be kept at the defined values by using the control system. Thus, the steady-state voltage-current characteristics mainly depend on the temperature. However, the previous studies of analytical models did not take into account the variation of the temperature. This study is focused on developing an analytical model of steady-state voltage-current-temperature characteristics of PEM fuel cells.

In this paper, the two-dimensional least square polynomial is used to analytically model the steady-state voltage-current-temperature characteristics of fuel cells. Firstly, a limited number of given steady-state voltage-current-temperature characteristics are computed by using electrochemical equations or acquired from experiment. Then, the coefficients in the proposed analytical model are computed by using the two-dimensional least square technique. Finally, the output steady-state voltage of fuel cells at arbitrary both current and temperature can be determined within the specified ranges of the current and the temperature.

Analytical Model

The proposed analytical model to describe the steady-state voltage-current-temperature characteristics can be defined as follows.

$$V_{out} = \sum_{j=0}^{p-1}\sum_{k=0}^{q-1} c_{jk}(I-\bar{I})^j (T-\bar{T})^k \quad (1)$$

$$\bar{I} = \sum_{j=1}^{N_I} I_j / N_I \quad (2)$$

$$\bar{T} = \sum_{j=1}^{N_T} T_j / N_T \quad (3)$$

where V_{out} represents the cell or stack voltage, (p-1) represents the number of the highest order with respect to the current, (q-1) represents the number of the highest order with respect to the temperature, c_{jk} represents the least square coefficients in the model, I represents the current, T represents the temperature, $\bar{I}$ represents the average value of the given current data, $\bar{T}$ represents the average value of the given temperature data, I_j represents the given current, N_I represents the number of the given current data, T_j represents the given temperature, and N_T represents the number of the given temperature data ($p \le N_I$ and $q \le N_T$).

For the proposed model, the good accuracy requires the large p and q. In this paper, p and q are equal to 15. The coefficients in model can be computed by using the two-dimensional least square technique (Xue, et. al. 2004)).

In order to evaluate the accuracy of the proposed analytical model, three kinds of computation errors are used in this paper. The square sum of absolute errors is defined by

$$SSAE = \sum_{j=1}^{N_{IT}} (V_{cj} - V_{gj})^2 \quad (4)$$

where N_{IT} denotes the number of the given voltage data, V_{cj} denotes the computed voltage, and V_{gj} denotes the given voltage.

The maximum absolute value of absolute errors is computed from

$$MAVAE = \max_{1 \le j \le N_{IT}} \{|V_{cj} - V_{gj}|\} \quad (5)$$

The maximum absolute value of relative errors is expressed by

$$MAVRE = \max_{1 \le j \le N_{IT}} \{\left|\frac{V_{cj} - V_{gj}}{V_{gj}}\right|\} \quad (6)$$

Applications

Application to Ballard Mark V PEM Fuel Cell

The given steady-state voltage-current-temperature characteristics for Ballard Mark V fuel cell are obtained from the computation (Xue, et. al. 2006), drawn in Figure 1.

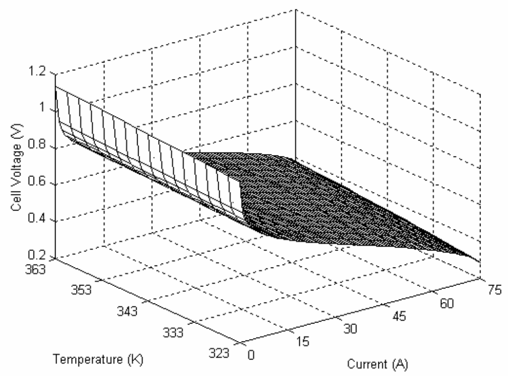


Figure 1. Given Steady-State Voltage-Current-Temperature Characteristics for Ballard Mark V Fuel Cell

Table 2 in the Appendix shows the computed least square coefficients. The steady-state voltage-current-temperature characteristics computed by using the proposed analytical model are illustrated in Figure 2. Figure 3 depicts the comparisons between the given and computed characteristics.

It can be seen from Figure 3 that the computed characteristics are considerable consistent with the given ones, for Ballard Mark V fuel cell.

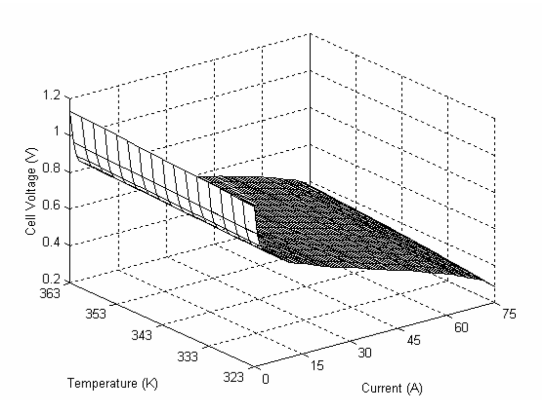
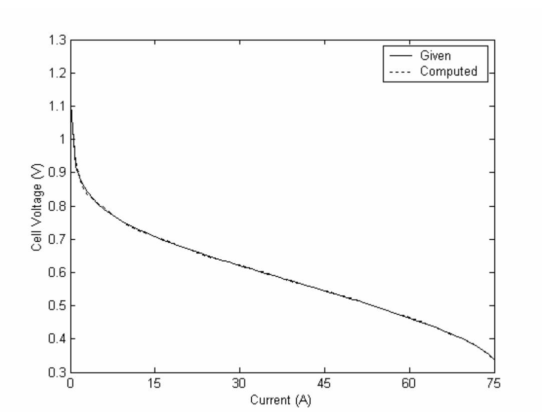
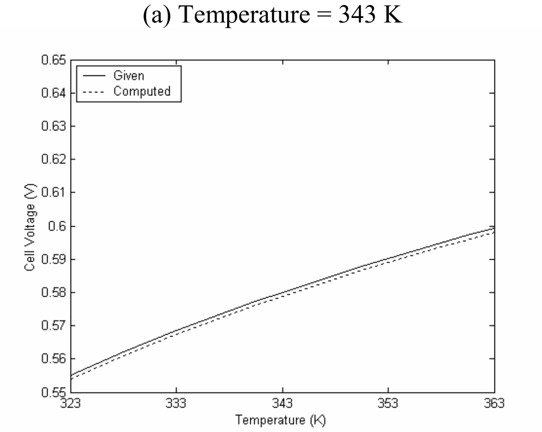


Figure 2. Computed Steady-State Voltage-Current-Temperature Characteristics for Ballard Mark V Fuel Cell



(a) Temperature = 343 K



(b) Current = 38 A

Figure 3. Comparisons between the Given and Computed Characteristics for Ballard Mark V Fuel Cell

Application to 500-W BCS PEM Fuel Cell Stack

The given steady-state voltage-current-temperature characteristics for 500-W BCS fuel cell stack are computed by using a group of electrochemical equations (Xue, et. al. 2006). They are shown in Figure 4.

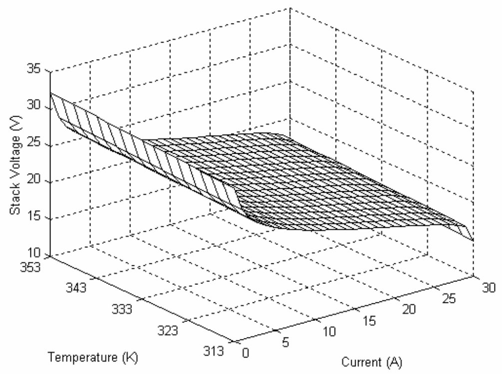


Figure 4. Given Steady-State Voltage-Current-Temperature Characteristics for 500-W BCS Fuel Cell Stack

Based on the given data in Figure 4, the least square coefficients are computed, as shown in Table 3 in the Appendix. Thus, the steady-state stack voltage can be computed at the arbitrary current and temperature. The computed steady-state voltage-current-temperature characteristics are depicted in Figure 5.

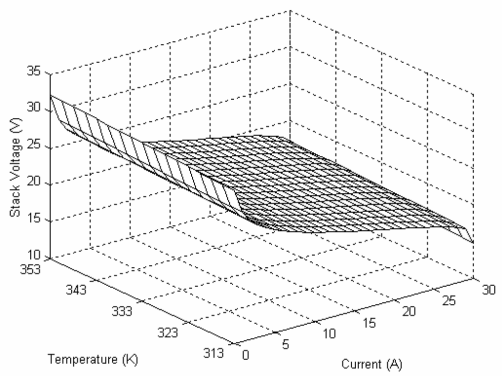
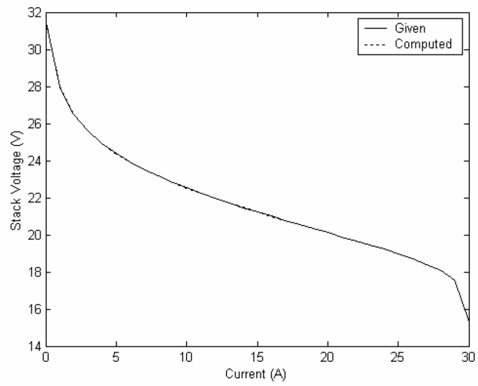
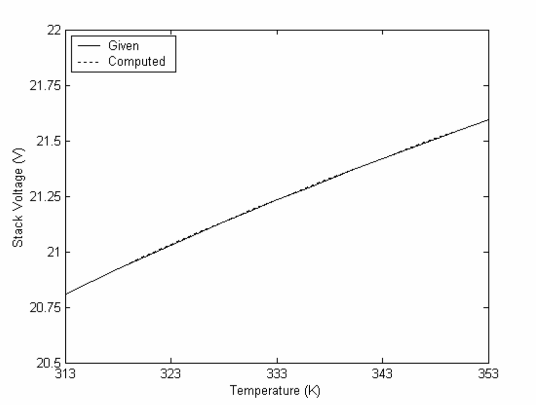


Figure 5. Computed Steady-State Voltage-Current-Temperature Characteristics for 500-W BCS Fuel Cell Stack

Fig. 6 illustrates the comparisons between the computed results and the given data. It can be observed that the computed characteristics are well consistent with the given ones.



(a) Temperature = 333 K



(b) Current = 15 A

Figure 6. Comparisons between the Computed and Given Stack Voltage for 500-W BCS Fuel Cell Stack

Application to Avista SR-12 PEM Fuel Cell Stack

Figure 7 shows the given steady-state voltage-current-temperature characteristics for Avista SR-12 PEM fuel cell stack, computed by using a number of electrochemical equations (Xue, et. al. 2006). From these given data, the coefficients in the model can be computed, as shown in Table 4 in the Appendix. Hence, the steady-state voltage-current-temperature characteristics computed by using the developed model are illustrated in Figure 8.

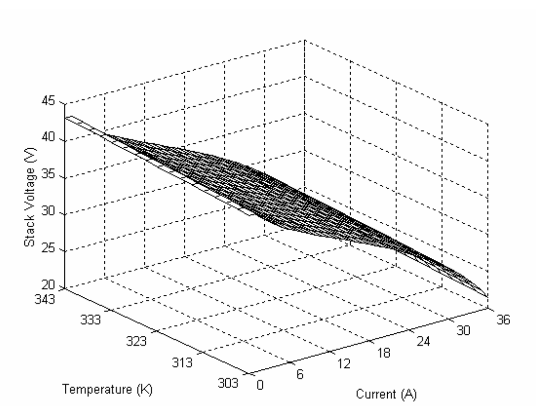


Figure 7. Given Steady-State Voltage-Current-Temperature Characteristics for SR-12 PEM Fuel Cell Stack

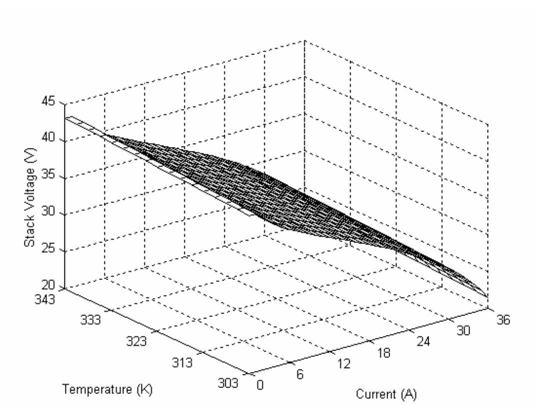
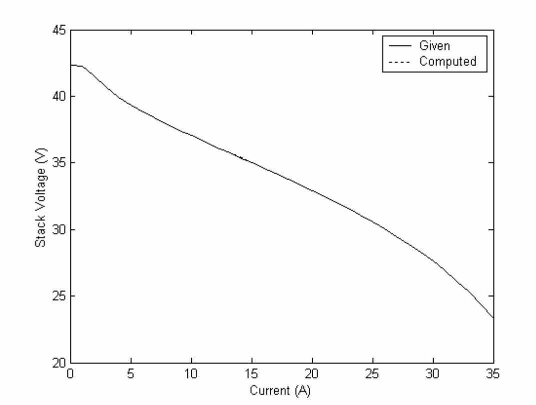
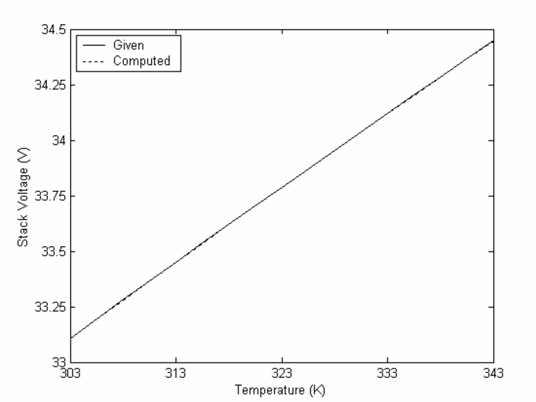


Figure 8. Computed Steady-State Voltage-Current-Temperature Characteristics for SR-12 PEM Fuel Cell Stack

At the same time, Figure 9 depicts the comparisons between the computed and given characteristics. It is clear that the computed results are excellent consistent with the given data.



(a) Temperature = 323 K



(b) Current = 18 A

Figure 9. Comparisons between the Computed and Given Characteristics for SR-12 PEM Fuel Cell Stack

Error Analysis

Three types of errors are used to indicate the accuracy of the proposed model. Table 1 indicates three types of errors for three applications. It can be seen that these errors are significantly small for such nonlinear characteristics. It validates that the proposed analytical model can be used to accurately describe the steady-state voltage-current-temperature characteristics for PEM fuel cells.

Table 1. Computation Errors for Three Kinds of PEM Fuel Cells

Error	**Case**		
	Ballard Mark V fuel cell	**500-W BCS fuel cell stack**	**SR-12 PEM fuel cell stack**
SSAE	0.1106922E-01	0.2020335E-01	0.1715789E-01
MAVAE	0.1757362E-01	0.1716747E-01	0.2196599E-01
MAVRE	0.6148383E-01	0.1165013E-02	0.1023359E-02

CONCLUSIONS

The analytical model of steady-state voltage-current-temperature for PEM fuel cells has been proposed in this paper. The proposed model is composed of the two-dimensional least square polynomial. The coefficients in the model are determined from a limited number of given steady-state voltage-current-temperature data, which are acquired from experiment or computed by using a group of electrochemical equations. The proposed model has been applied to three kinds of PEM fuel cells successfully. For anyone of three kinds of PEM fuel cells, the computed results from the proposed model are agreement well with the given data. Thus, it demonstrated that the proposed analytical model can be used to accurately model the steady-state voltage-current-temperature characteristics of PEM fuel cells. As a result, this paper offers a valuable approach for design of the power conditioning units, design of simulation of fuel cell stack systems, and design of system controllers.

ACKNOWLEDGEMENT

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APPENDIX

Table 2. Coefficients (c_{jk}) in the Proposed Model for Ballard Mark V Fuel Cell

j \ k	0	1	2	3	4	5	6	7
0	.58157085E+00	.10723016E-02	-.67671937E-05	.47476007E-07	.10454232E-09	-.14025002E-10	-.10938497E-10	.37647276E-12
1	-.55035627E-02	.18990680E-04	-.20704977E-06	.18369811E-08	-.21277426E-09	-.61353258E-11	.42390592E-11	.39227077E-13
2	.86467514E-04	.45338839E-06	-.57768663E-09	.28591835E-10	-.29937029E-10	-.26073243E-12	.60984002E-12	-.20117790E-16
3	.11736083E-04	.35079901E-07	-.53740467E-10	-.47362917E-11	.24926641E-11	.66825885E-13	-.51043617E-13	-.20965671E-15
4	-.11729955E-05	-.34929558E-08	-.29731390E-11	-.64433444E-13	.21051154E-12	.15767875E-14	-.42192733E-14	-.59504828E-17
5	-.85868710E-07	-.24428733E-09	.25660771E-12	.29464254E-13	-.13677642E-13	-.45328400E-15	.27717498E-15	.21390737E-17
6	.59961847E-08	.17936713E-10	.87552470E-14	-.42567240E-15	-.67369011E-15	.62044592E-17	.13241888E-16	-.78654641E-19
7	.24900690E-09	.70823644E-12	-.62706811E-15	-.80007336E-16	.33422546E-16	.13451090E-17	-.67064264E-18	-.80919904E-20
8	-.14313196E-10	-.42839796E-13	-.15769137E-16	.19650250E-17	.12385825E-17	-.34903411E-19	-.23895903E-19	.35929319E-21
9	-.34773533E-12	-.98813805E-15	.73138687E-18	.10229911E-18	-.38898065E-19	-.18100569E-20	.77357784E-21	.12166518E-22
10	.17362198E-13	.52001357E-16	.16945563E-19	-.29668723E-20	-.13047614E-20	.55241395E-22	.24869865E-22	-.54684620E-24
11	.23055667E-15	.65455557E-18	-.40320435E-21	-.60943536E-22	.21350019E-22	.11097233E-23	-.42097423E-24	-.78891170E-26
12	-.10349555E-16	-.31018480E-19	-.95297117E-23	.19233698E-23	.71197860E-24	-.36490448E-25	-.13498566E-25	.35518913E-27
13	-.58432572E-19	-.16568008E-21	.84614742E-25	.13673942E-25	-.44528475E-26	-.25305398E-27	.87052898E-28	.18540152E-29
14	.24084549E-20	.72246239E-23	.21128748E-26	-.45583479E-27	-.15391474E-27	.87200206E-29	.29160189E-29	-.84024510E-31

j \ k	8	9	10	11	12	13	14	
0	.11084910E-12	-.24945329E-14	-.53116302E-15	.67069032E-17	.12013213E-17	-.63750322E-20	-.10290617E-20	
1	-.40153198E-13	-.91023112E-16	.18699783E-15	.27721145E-19	-.41632659E-18	.96076902E-22	.35282111E-21	
2	-.55461632E-14	.10669838E-16	.24922192E-16	-.43217652E-19	-.53962779E-19	.49706355E-22	.44789166E-22	
3	.48184483E-15	-.91959395E-18	-.22457759E-17	.56594368E-20	.50075827E-20	-.72899545E-23	-.42494682E-23	
4	.37161993E-16	-.25786581E-19	-.16144787E-18	.17105280E-21	.33888077E-21	-.22730394E-24	-.27380549E-24	
5	-.25911427E-17	-.19079762E-21	.11983217E-19	-.18321108E-22	-.26561770E-22	.28522045E-25	.22439495E-25	
6	-.11237527E-18	.52324259E-21	.46806143E-21	-.15265091E-23	-.94228312E-24	.15649977E-26	.73241738E-27	
7	.62066994E-20	.16223303E-22	-.28466418E-22	.53549563E-26	.62689681E-25	-.33374954E-28	-.52696198E-28	
8	.19702372E-21	-.19387400E-23	-.79331988E-24	.49492865E-26	.15407566E-26	-.46748477E-29	-.11554087E-29	
9	-.70981335E-23	-.33327193E-25	.32325288E-25	.28849149E-28	-.70790120E-28	.71401830E-32	.59244120E-31	
10	-.20222801E-24	.27907145E-26	.80125033E-27	-.68054100E-29	-.15286418E-29	.62185004E-32	.11249148E-32	
11	.38332916E-26	.24310973E-28	-.17348774E-28	-.29870946E-31	.37804693E-31	.84067754E-35	-.31513704E-34	
12	.10945213E-27	-.17662186E-29	-.43248333E-30	.42084833E-32	.82257781E-33	-.37771549E-35	-.60323855E-36	
13	-.78702466E-30	-.60413690E-32	.35415707E-32	.83781139E-35	-.76820720E-35	-.35648080E-38	.63799303E-38	
14	-.23708968E-31	.41140735E-33	.94030114E-34	-.96632209E-36	-.17956675E-36	.85732004E-39	.13222290E-39	

Table 3. Coefficients (c_{jk}) in the Proposed Model for 500-W BCS Fuel Cell Stack

j \ k	0	1	2	3	4	5	6	7

0	.21234356E+02	.19373381E-01	-.77205658E-04	.62864949E-06	.17118084E-08	-.11460798E-08	-.99110056E-10	.11469647E-10
1	-.24006006E+00	.19798195E-03	-.52248356E-05	.70862921E-07	-.90709769E-08	-.67309953E-09	.18298937E-09	.61626174E-11
2	.22424211E-02	.16639662E-04	.10097408E-06	-.12221367E-07	-.57346130E-08	.31660627E-09	.10419232E-09	-.32539904E-11
3	.62745093E-03	.97948119E-07	-.18925221E-07	-.21953289E-08	.11483710E-08	.49499173E-10	-.23141069E-10	-.46015349E-12
4	.10180914E-03	-.82002769E-07	-.63978166E-08	.10977420E-08	.36472957E-09	-.26919622E-10	-.70197400E-11	.27005117E-12
5	-.41747476E-04	-.34496069E-07	.81058889E-09	.57593887E-10	-.45537108E-10	-.13448046E-11	.89893865E-12	.12758522E-13
6	-.31651058E-05	.40693901E-08	.15186455E-09	-.30677829E-10	-.93204481E-11	.74000293E-12	.18374442E-12	-.73516966E-14
7	.80465045E-06	.65660611E-09	-.14205828E-10	-.71777428E-12	.77098898E-12	.17613107E-13	-.15084467E-13	-.17199353E-15
8	.50505198E-07	-.66161682E-10	-.18433624E-11	.39058010E-12	.11741460E-12	-.93281308E-14	-.23390808E-14	.92092246E-16
9	-.76251248E-08	-.64838198E-11	.11669899E-12	.45246531E-14	-.62471450E-14	-.11730256E-15	.12188612E-15	.11801815E-17
10	-.40204785E-09	.57145137E-12	.11962512E-13	-.25290889E-14	-.77475874E-15	.59865058E-16	.15495543E-16	-.58765267E-18
11	.34250242E-10	.30335694E-13	-.44804811E-15	-.13965489E-16	.23886765E-16	.38298403E-18	-.46617242E-18	-.39630370E-20
12	.15632781E-11	-.24379731E-14	-.39452573E-16	.81398212E-17	.25661213E-17	-.19089591E-18	-.51343441E-19	.18628667E-20
13	-.59345078E-13	-.55896561E-16	.64862642E-18	.16804659E-19	-.34597513E-19	-.48728759E-21	.67635236E-21	.51736120E-23
14	-.23252925E-14	.41835620E-17	.51847609E-19	-.10348704E-19	-.33616635E-20	.24036578E-21	.67151112E-22	-.23313573E-23

j \ *k*	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
0	.93508118E-12	-.54919855E-13	-.45200658E-14	.12510392E-15	.10467012E-16	-.10804255E-18	-.91663155E-20	
1	-.16850732E-11	-.27353687E-13	.76687714E-14	.58286559E-16	-.16815589E-16	-.47610159E-19	.14119253E-19	
2	-.89953527E-12	.15784880E-13	.39515020E-14	-.35998134E-16	-.84781497E-17	.30999947E-19	.70153237E-20	
3	.21075672E-12	.20633855E-14	-.95369628E-15	-.44202165E-17	.20834007E-17	.36190324E-20	-.17445972E-20	
4	.62316552E-13	-.12885728E-14	-.27793553E-15	.29029397E-17	.60179330E-18	-.24764987E-20	-.50092252E-21	
5	-.81352807E-14	-.58060861E-16	.36726479E-16	.12580163E-18	-.80138369E-19	-.10392583E-21	.67055682E-22	
6	-.16488897E-14	.34836471E-16	.73972648E-17	-.78078139E-19	-.16074562E-19	.66351668E-22	.13412694E-22	
7	.13622914E-15	.79939454E-18	-.61478910E-18	-.17610205E-20	.13414710E-20	.14742641E-23	-.11224581E-23	
8	.21086401E-16	-.43432712E-18	-.94840668E-19	.96998002E-21	.20644049E-21	-.82209685E-24	-.17246317E-24	
9	-.11009338E-17	-.56023399E-20	.49715227E-20	.12544281E-22	-.10853338E-22	-.10637811E-25	.90842262E-26	
10	-.13991583E-18	.27592872E-20	.62991220E-21	-.61414108E-23	-.13721081E-23	.51917086E-26	.11469149E-26	
11	.42158639E-20	.19176525E-22	-.19058148E-22	-.43564382E-25	.41636617E-25	.37362545E-28	-.34866038E-28	
12	.46360257E-21	-.87069945E-23	-.20872866E-23	.19310458E-25	.45471724E-26	-.16279782E-28	-.38014294E-29	
13	-.61268878E-23	-.25461819E-25	.27731194E-25	.58573934E-28	-.60634230E-28	-.50722958E-31	.50800897E-31	
14	-.60578017E-24	.10845275E-25	.27260794E-26	-.23963805E-28	-.59373185E-29	.20144952E-31	.49630079E-32	

Table 4. Coefficients (c_{jk}) in the Proposed Model for SR-12 PEM Fuel Cell Stack

j \ *k*	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
0	.33786986E+02	.33405498E-01	-.25450369E-04	.17875731E-06	.36754298E-08	.78049303E-09	-.64949711E-10	-.90604055E-11
1	-.41931416E+00	-.12083925E-03	-.15883303E-05	.22943936E-07	.60956375E-08	-.24402966E-09	-.12539922E-09	.21779132E-11
2	-.32766824E-02	.94219475E-05	.20961467E-08	-.33783783E-08	.39616381E-09	.35253522E-10	-.17014823E-10	-.16127966E-12
3	-.27951276E-03	-.42459051E-06	.10837783E-07	-.74298559E-09	-.59580970E-09	.17143206E-10	.11735645E-10	-.16897985E-12
4	-.27057161E-04	-.41123976E-07	-.21993055E-09	.24882758E-09	-.23422434E-10	-.43166045E-11	.82500790E-12	.37540063E-13
5	.79249905E-06	.58867505E-08	-.26243621E-09	.13659764E-10	.14062457E-10	-.33555073E-12	-.27378900E-12	.35232370E-14
6	.47629992E-06	.92320968E-09	.13240792E-12	-.53640299E-11	.57978686E-12	.10158112E-12	-.16731233E-13	-.93498024E-15
7	-.27038957E-07	-.11620332E-09	.27279571E-11	-.11705881E-12	-.14295234E-12	.31236297E-14	.27507986E-14	-.34970897E-16
8	-.37411893E-08	-.57806997E-11	.25306167E-13	.51157828E-13	-.60756701E-14	-.10018348E-14	.15776276E-15	.94132500E-17
9	.24262958E-09	.92649322E-12	-.14054664E-13	.52000516E-15	.72058710E-15	-.15087225E-16	-.13690919E-16	.17827934E-18
10	.12216630E-10	.90129691E-14	-.19615710E-15	-.24232826E-15	.30350431E-16	.48337562E-17	-.74217460E-18	-.45954712E-19

11	-.97691343E-12	-.34251567E-14	.35290375E-16	-.11729569E-17	-.17725204E-17	.36578545E-19	.33242372E-19	-.44979107E-21
12	-.79420401E-14	.43666945E-16	.56052980E-18	.55904474E-18	-.71962417E-19	-.11286787E-19	.16946717E-20	.10818919E-21
13	.13839124E-14	.45924305E-17	-.34368177E-19	.10657423E-20	.16944014E-20	-.35124272E-22	-.31379828E-22	.44357353E-24
14	-.18851237E-16	-.11787031E-18	-.56191265E-21	-.50114080E-21	.65248176E-22	.10204818E-22	-.14975210E-23	-.98436627E-25
j* \ *k	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
0	.34981139E-12	.46406358E-13	-.83268224E-15	-.10832807E-15	.70310074E-18	.94255067E-19	.52453044E-22	
1	.11739533E-11	-.93921692E-14	-.54579191E-14	.19406293E-16	.12187907E-16	-.15363407E-19	-.10379594E-19	
2	.20843563E-12	.33971677E-15	-.11080079E-14	-.25650761E-18	.26713582E-17	-.38500738E-22	-.23856458E-20	
3	-.10726569E-12	.80367811E-15	.48966741E-15	-.18125165E-17	-.10784511E-17	.15485719E-20	.90887081E-21	
4	-.94491966E-14	-.17134150E-15	.48552716E-16	.38494346E-18	-.11481903E-18	-.33216602E-21	.10134317E-21	
5	.24796468E-14	-.17597530E-16	-.11235461E-16	.41171129E-19	.24599005E-19	-.36161668E-22	-.20635400E-22	
6	.17852820E-15	.44103651E-17	-.88459376E-18	-.10096251E-19	.20482052E-20	.88115455E-23	-.17842638E-23	
7	-.24690028E-16	.18227543E-18	.11108521E-18	-.43885976E-21	-.24189535E-21	.39324843E-24	.20208612E-24	
8	-.16097122E-17	-.44935309E-19	.77811448E-20	.10358409E-21	-.17747997E-22	-.90796788E-25	.15313110E-25	
9	.12169508E-18	-.95927054E-21	-.54346098E-21	.23557451E-23	.11767915E-23	-.21386922E-26	-.97900870E-27	
10	.73589352E-20	.22093110E-21	-.34981227E-22	-.51149510E-24	.78953462E-25	.44961043E-27	-.67655018E-28	
11	-.29254468E-21	.24723172E-23	.12965134E-23	-.61475821E-26	-.27914347E-26	.56255970E-29	.23124359E-29	
12	-.16476934E-22	-.52284199E-24	.77391419E-25	.12144916E-26	-.17333269E-27	-.10699225E-29	.14777158E-30	
13	.27346690E-24	-.24703994E-26	-.12029397E-26	.61878535E-29	.25754786E-29	-.56880330E-32	-.21246389E-32	
14	.14351900E-25	.47770676E-27	-.66799650E-28	-.11127269E-29	.14872226E-30	.98214125E-33	-.12628716E-33	

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