# EXPERIMENTAL VERIFICATION OF THE END-EFFECT WEIBULL MODEL

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## Introduction

Tensile failure is inherently probabilistic. Microscopic flaws and crystalline misalignments cause the fibers to fail below their intrinsic tensile strength [1]. As a result, the actual strength at which a given fiber will fail depends on both the frequency and severity of these flaws. Therefore, statistical analysis of tensile failure data should provide insight into the distribution of flaws within the fiber.

Because tensile failure data do not conform to rigid statistical distributions, a flexible distribution that can be altered by the data itself is required. The Weibull distribution is the preferred form for analyzing most failure data [2-5]. However, even the flexible Weibull distribution cannot separate the failures that result from true flaws from those that are artifacts of the tensile test itself. These "clamp effects" are analyzed in detail by Phoenix and Sexsmith [6]. In building upon this work, Stoner [7] developed the end-effect Weibull model, in which distinct Weibull distributions were used to characterize failures from true flaws and from artifacts in carbon fibers. This model was subsequently used to trace the propagation of flaws from precursor fibers into carbon fibers [8].

Although the end-effect Weibull model accurately represented the data upon which it was based, no attempts were made to use the Weibull parameters as predictors of tensile failure at other gauge lengths. This paper describes the application of the end-effect Weibull model to predict tensile strengths of Kevlar fibers at gauge lengths other than those used to evaluate the Weibull parameters. The same approach is directly applicable to carbon fibers.

## End-Effect Weibull Model

Previous research [5,8] has shown that the simple Weibull model cannot account for failure artifacts introduced during tensile testing by the non-uniform stress distribution that occurs at the fiber-glue interface where the fiber is mounted to the testing tab. Detailed descriptions of the end-effect model and the nature of end effects have been published previously by Stoner et al. [8]. Although all fibers are subject to end effects, longer fibers are more likely to contain a fatal flaw, making end-effect failure less likely.

The total survival relationship becomes

$$S = \exp(-L\left(\frac{\sigma}{\sigma_{oF}}\right)^{m_F} - \left(\frac{\sigma}{\sigma_{oE}}\right)^{m_E})$$

while the probability of failure is given by

$$F = 1 - \exp(-L\left(\frac{\sigma}{\sigma_{oF}}\right)^{m_F} - \left(\frac{\sigma}{\sigma_{oE}}\right)^{m_E})$$

The above can be used to evaluate the four empirical Weibull parameters ($m_F$, $m_E$, $\sigma_{oE}$, $\sigma_{oF}$). In this case, $m_F$ and $\sigma_{oF}$ represent the shape and scale parameters, respectively, for failures resultimg from true flaws, while $m_E$ and $\sigma_{oE}$,represent the shape and scale parameters for failures caused by end effects. However, this evaluation is more complex than the simple Weibull model because the equation cannot be linearized. These parameters in these non-linear equations were evaluated using maximum-likelihood estimation. Essentially, maximum likelihood theory evaluates the probability that a given set of tensile strengths would have resulted from a specific set of Weibull parameters. The observed failure distributions are provided in the above equation, while the parameters are systematically varied to maximize the likelihood that the observed failure distribution resulted from the given parameters.

## Experimental

As-received Kevlar-29 samples were obtained from the E. I. DuPont de Nemours Corporation. The single filament testing procedure used in this study followed the procedure described in ASTM Standard D-3379-75 [9]. Fibers were mounted on testing tabs using Hughes Epoxy 220 and placed in a 65°C drying oven for 24 hours. Fiber diameters. were measured using a laser diffraction technique with a 0.95 mW helium-neon red laser. This technique has been found to be accurate to within 0.1 μm. A minimum of 150 samples were tested at each of three gauge lengths (10, 25 and 40 mm).

The tensile failure data obtained from these three gauge lengths were used to determine the four Weibull parameters. Next, 175 filaments were tested using a 5 mm gauge length. The results of this test were compared to predictions made by applying the Weibull parameters determined from the 10, 25, and 40 mm tests.

## Results and Discussion

Table 1 provides the shape and scale parameters for the Kevlar fibers as determined from maximum likelihood analysis of the 10, 25, and 40 mm data. These data were then applied to the 5 mm samples. Figure 1 shows that the same values of the empirical parameters provided a highly accurate prediction, within five percent of observed failure values for the 5 mm tests. The model can also be deconstructed to separate the influence of true flaws from end effects, since independent Weibull shape and scale parameters are determined for the flaw and end effect distributions. As expected, end effects resulted in a relatively small amount of failures for large gauge length samples and substantially more as gauge length decreased. These results compare favorably with those obtained using the more complicated and data intensive stochastic model proposed by Wagner [10], without requiring consideration of diameter variations between individual filaments.

These results have two significant consequences. First, they provide experimental validation of the end-effect Weibull model as evidenced by the ability of the model to predict failures at a gauge length outside its initial range, without needing to consider the physics of the "clamp effects". Secondly, the model shows promise as offering a means of predicting the functional tensile strength of fibers in composites at a wide variety of gauge lengths with relatively minimal amounts of required data.

## Conclusions

This work has shown that the end-effect Weibull model provides an exceptional fit of the tensile failure data upon which its parameters are based. More importantly, the end-effect Weibull model accurately predicted the tensile failure rates at a gauge length outside the data range used to determine the parameters. This serves to both support the validity of the end-effect model and to provide a means of predicting failure rates at a wide range of gauge lengths. Finally, the data showed that end effects became less significant as gauge length increased.

## References


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Table 1. Shape and Scale Parameters for the End-Effect Weibull Model based upon Data from 10, 25, and 40 mm Gauge Length Samples.

| | |
|---|---|
| $m_F$ | 4.6091 |
| ,$\sigma_{oF}$ | 3.4452 |
| $m_E$ | 5.2261 |
| $\sigma_{oE}$ | 1.5880 |

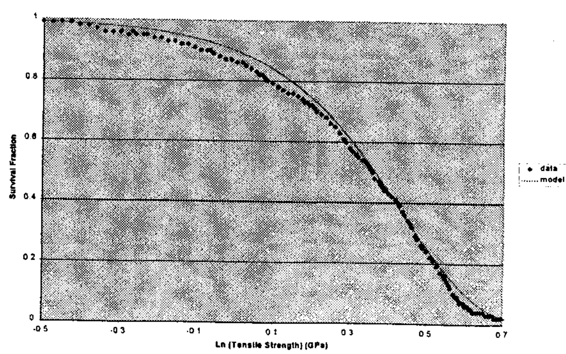


Figure 1. End-Effect Weibull Model Applied as a Predictor of Tensile Failure at 5 mm Gauge Length Based upon Parameters Determined from 10, 25, and 40 mm Trials.