

Julius Jortner

Jortner Research & Engineering, Inc
Cloverdale, Oregon 97112-0219, USA

Estimation of thermoelastic properties of 3D carbon-carbons using micromechanical analysis is problematic because microcracks interrupt the microstructure [1]. To avoid the difficulties of dealing explicitly with microcracks, several analysts have (with some practical success) treated aggregates as if uncracked but made of materials of reduced stiffness. For example, before computing properties of polycrystalline graphites, Smith *et al* [2] adjust downwards the c-axis extensional and basal-plane shear stiffnesses of the constituent grains. Similar *ad hoc* approaches have been applied to 3D carbon-carbon properties [3,4]; in [4], "efficiency" factors ranging from 0 to 1 are applied as multipliers to the transverse and shear stiffnesses estimated for uncracked yarn bundles; in [3] the factors are defined in relation to hypothetical springs inserted between composite phases, but the effect is essentially the same as in [4]. Although intuitively appealing, these attempts lack *a priori* knowledge of the degree of stiffness degradation. Thus, the efficiency factors usually are quantified by fitting analytical predictions to measured stiffnesses or thermal expansions. Here, we offer a supplementary way to quantify extensional efficiencies, using observed effects of micromechanical residual stresses.

Feldman [5] reports that strips excised with planar cuts from 3D carbon-carbon blocks tend to curl. If thin enough, the strip can be viewed as a bi-material construct of one layer each of materials "a" and "b" (Fig. 1), where "a" contains yarns running across the width of the strip

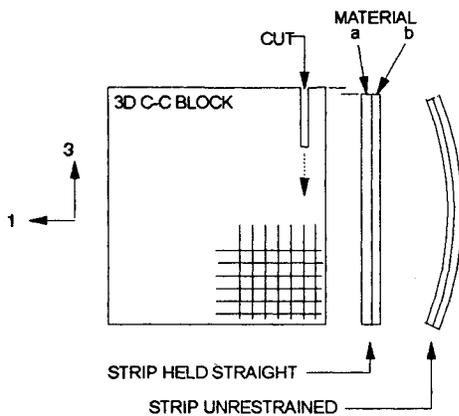


Fig. 1. Schematic of strip curvature.

and "b" contains yarns parallel to the length of the strip. Such strips curl with "a" on the concave side, and they straighten when heated [5]. The analysis below suggests this curvature can be a measure of the effective *in-situ* stiffnesses of the composite's microcracked constituents.

The 3D composite may be viewed (Fig. 2) as comprising four regions (labeled I-IV) for purposes of estimating properties in the direction of the strip length (the 3-direction). The relevant properties of the regions are estimated from the lengths, stiffnesses, and thermal expansions of the constituent phases (unidirectional fiber-bundle composite and the matrix pockets) assuming continuity of stress across the interface between the two phases. Poisson's ratios are assumed negligible.

$$E_I = \eta \left[\frac{l_{T_I}}{E_T} + \frac{l_{M_I}}{E_M} \right]^{-1} \quad \alpha_I = \alpha_T l_{T_I} + \alpha_M l_{M_I}$$

$$E_{III} = \eta \left[\frac{l_{T_{III}}}{E_T} + \frac{l_{M_{III}}}{E_M} \right]^{-1} \quad \alpha_{III} = \alpha_T l_{T_{III}} + \alpha_M l_{M_{III}}$$

$$E_{II} = \eta E_T \quad \alpha_{II} = \alpha_T$$

$$E_{IV} = E_L \quad \alpha_{IV} = \alpha_L$$

In these expressions, E is the Young's modulus, α is the thermal expansion coefficient, l is the relevant length fraction, and the efficiency factor η can take values between 1 and 0 depending on the extent of degradation by microcracks. The subscripts M, T, and L refer respectively to matrix pocket material and to the transverse and longitudinal directions of fiber bundles.

The effective thermoelastic properties of the 3D composite as a whole (E_C, α_C), and of layers a and b

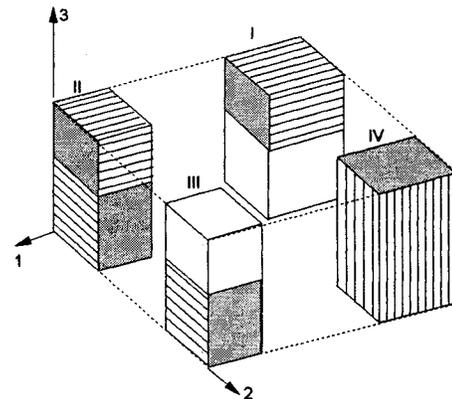


Fig. 2. Exploded view of 3D unit cell showing four regions considered in the analysis.

within it ($E_a, E_b, \alpha_a, \alpha_b$), are then estimated from the properties and relative dimensions of the regions, for loadings giving equal 3-direction strain in each region. The equations are of the form: $E_C = E_a l_a + E_b l_b$ and $\alpha_C = \frac{1}{E_C} [\alpha_a E_a l_a + \alpha_b E_b l_b]$.

Within a 3D block at a temperature T different from the stress-free temperature T_0 there are residual strains; for example, in component "i" the residual strain in the 3-direction is $\epsilon_{Ri} = (\alpha_C - \alpha_i)(T - T_0)$ with a residual stress of approximately $E_i \epsilon_{Ri}$. Then, a thin strip cut from the 3D block will take on curvature caused by the residual stresses experienced by layers "a" and "b" while in the block.

First, imagine the strip cut from the block but held straight in a way that allows the strip to change length. The mechanical strains in phases a and b will be:

$$\epsilon_{Sa} = (\alpha_S - \alpha_a)(T - T_0) \quad \text{and}$$

$$\epsilon_{Sb} = (\alpha_S - \alpha_b)(T - T_0) \quad \text{where}$$

$$\alpha_S = (\alpha_a E_a r + \alpha_b E_b (1 - r)) (E_a r + E_b (1 - r))^{-1}$$

is the effective thermal expansion of the strip, which may differ from that of the composite because the ratio of phases "a" and "b" may be different in the strip than in the block; r is the ratio of the thickness of phase "a" to the total strip thickness. The strip as a whole changes length, when excised, by a strain $\epsilon_S = (\alpha_S - \alpha_C)(T - T_0)$.

Now, allow the strip to bend. The curvature K is (as estimated from simple beam theory, based on equilibrium of forces and of moments, and the assumption that initially plane sections of a beam remain plain):

$$K \equiv 1/R = C[D - B^2/A] \quad \text{where}$$

$$A \equiv E_a r + E_b (1 - r) \quad B \equiv \frac{1}{2}(E_a r^2 + E_b (1 - r^2))$$

$$C \equiv \frac{1}{2}[E_a \epsilon_{Sa} r^2 + E_b \epsilon_{Sb} (1 - r^2)] \quad D \equiv \frac{1}{3}[E_a r^3 + E_b (1 - r^3)]$$

We illustrate the analysis for the following inputs:

$E_M = E_T = .04 E_L$, thermal expansions (in microstrain per deg C) $\alpha_M = 4.1$, $\alpha_T = 8.4$, $\alpha_L = 0$, and 3D unit dimensions $l_{1f} = l_{1u}$, $l_{2f} = 1.5 l_{2u}$ and $l_{Tf} = 1.5 l_{Mf}$; the stress-free temperature is 1100 C, as estimated in [5]; the strip is 25 mm long and .25 mm thick.

Estimated length changes and curvatures at 20 C are shown in Figures 3 and 4., as functions of η , the microcrack-effect factor, and r , the thickness fraction of material "a". Length change and curvature are seen to be fairly sensitive functions of η and r .

These trends encourage the exploration of strip-curvature experiments to quantify η , as a potentially more direct and economical method than estimation from comparisons between predicted and measured composite stiffnesses and expansions.

References

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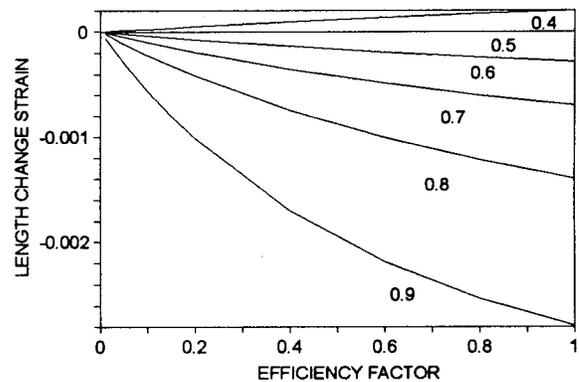


Fig. 3. Length change vs. η and r .

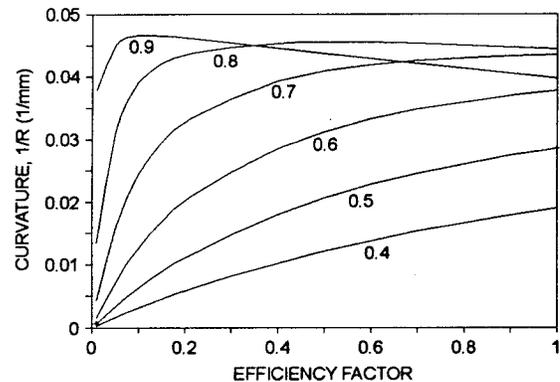


Fig. 4. Curvature vs. η and r .